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SHEAR-INDUCED RESUSPENSION IN A COUETTE DEVICE

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Abstract—The viscous resuspension of an initially settled bed of particles resulting from the application of a laminar shear flow was studied experimentally in a narrow-gap Couette device. The measured height of the resuspended layer was found to be in excellent agreement with that predicted theoretically using a model developed by Leighton & Acrivos, in which the downward gravitational particle flux is balanced by a corresponding upward flux due to shear-induced particle diffusion.

Key Words: resuspension, suspensions

1. INTRODUCTION

When a fluid flows past an initially settled bed of heavy, non-Brownian particles, at least part of the sediment layer will resuspend (even under laminar flow conditions) and the flowing suspension will acquire a non-uniform concentration profile. This phenomenon, termed viscous resuspension, was first reported by Gadala–Maria (1979) and was further investigated by Leighton & Acrivos (1986, 1987a,b) and by Schaflinger *et al.* (1990). According to Leighton & Acrivos (1987a,b), a test particle in a concentrated suspension being sheared undergoes a random walk as it interacts with neighboring particles (whose positions are also random) and thereby acquires a net drift velocity from regions of high particle concentration to low and from regions of high shear to low. The resulting shear-induced particle flux opposes that due to gravity and a steady-state particle concentration profile is attained under the proper conditions. Leighton & Acrivos (1986), hereafter referred to as LA1, then developed a mathematical model which they tested by measuring the height of a suspension flowing in a plane Couette device when both the gravitational and diffusive fluxes were parallel to the plane of shear. The agreement reported in LA1 between the experimental results and the theoretical predictions was excellent, even though the latter entailed *ab initio* calculations without the use of adjustable parameters.

In this work, we shall report the results of a theoretical and experimental investigation of such a viscous resuspension phenomenon in a narrow gap Couette device. But unlike the case treated in LA1, where the shear stress was constant and the diffusive flux was parallel to the plane of shear, in the present situation the shear rate is (approximately) constant across the gap and the diffusive flux is normal to the plane of shear. Thus, the shear-induced particle diffusion due to gradients in the shear rate is negligible, and hence diffusion due to the gradients in particle concentration can be treated as in LA1 using the results presented by Leighton & Acrivos (1987b), hereafter referred to as LA2.

The experimentally determined resuspension height was found to be in excellent agreement with that predicted theoretically, in spite of the absence of adjustable parameters in the model.

2. THEORETICAL ANALYSIS

Consider a suspension of heavy spherical particles of uniform size in a narrow Couette gap undergoing shear (figure 1). As in LA1, the downward flux of particles due to gravity is balanced by an upward particle flux due to the gradients in the particle concentration. (In this case, $\dot{\gamma}$, the shear rate across the gap is apprcximately uniform so there is no particle flux due to gradients in the shear rate.)

We begin by setting the particle velocity equal to the Stokes settling velocity multiplied by the hindrance function $f(\phi)$, which accounts for the influence of all the other particles in the suspension. The downward gravitational flux is therefore expressed as

$$N_s = \frac{2}{9}\phi f(\phi)\frac{a^2 g\,\Delta\rho}{\mu_0}, \qquad [1]$$

where ϕ is the particle volume fraction, a is the particle radius, g is the gravitational constant, $\Delta\rho$ is the difference between the particle density and that of the fluid and μ_0 is the suspending fluid viscosity. As in LA1, we approximate $f(\phi)$ by means of

$$f(\phi) = \frac{1-\phi}{\mu_r}, \qquad [2]$$

where μ_r is the relative viscosity of the suspension given by Leighton (1985) as

$$\mu_r = \frac{\mu(\phi)}{\mu_0} = \left[1 + \frac{1.5\phi}{1 - \dfrac{\phi}{\phi_0}}\right]^2, \qquad [3]$$

with ϕ_0 being the concentration of the settled layer (taken here as equal to 0.58).

The shear-induced particle flux in unidirectional shear flows was studied in LA2, along the directions parallel (r) and perpendicular (z) to the plane of shear, using the following constitutive relations:

$$N_{\parallel} = -D_{\parallel}\frac{d\phi}{dr} - D'\frac{d\tau}{dr} \qquad [4a]$$

and

$$N_{\perp} = -D_{\perp}\frac{d\phi}{dz} - D''\frac{d\dot{\gamma}}{dz}, \qquad [4b]$$

where $D_{\parallel}$ is the shear-induced diffusivity along the plane of shear when the shear stress τ is kept constant, while $D_{\perp}$ is the shear-induced diffusivity normal to the plane of shear at constant shear rate $\dot{\gamma}$. As shown in LA2 there appears to be no significant difference between $D_{\parallel}$ and $D_{\perp}$ when

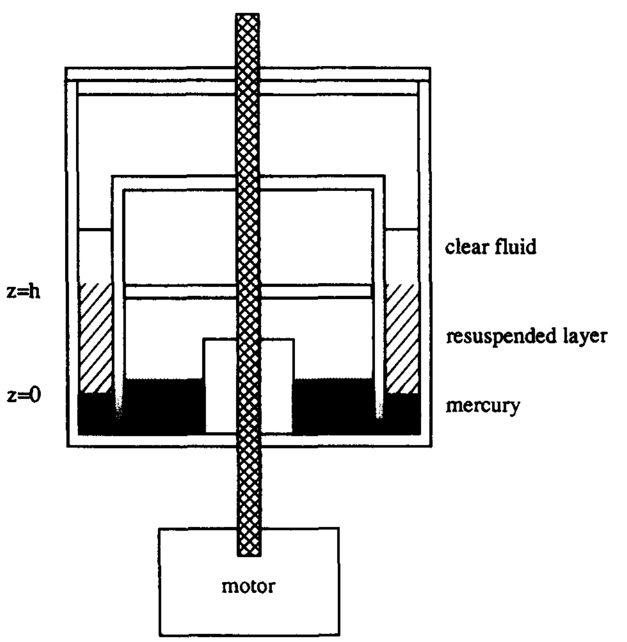

Figure 1. Schematic of the resuspension in a Couette gap and the experimental setup.

$\phi < 0.5$, both of which we shall denote here by D; moreover, as in LA1, we shall represent the dependence of the dimensionless effective diffusion coefficient, $\hat{D} = D/(\dot{\gamma}a^2)$, on ϕ by means of

$$\hat{D} \approx \tfrac{1}{3}\phi^2(1 + \tfrac{1}{2}e^{8.8\phi}). \qquad [5]$$

Chapman & Leighton (1991) further confirmed this correlation for shear-induced diffusion in the plane of shear. As mentioned earlier, the diffusive flux in our experiment is normal to the plane of shear, so that, at steady state, $N_\perp$ balances the downward gravitational flux N_s. Consequently, on equating equation [1] and [4b] with constant $\dot{\gamma}$ and integrating we obtain:

$$h = \frac{9}{2}\frac{\mu_0\dot{\gamma}}{g\Delta\rho}\int_0^{\phi(0)}\frac{\hat{D}(\phi)}{f(\phi)\phi}\,d\phi \qquad [6]$$

and

$$\textstyle\int_0^h \phi\,dz = \frac{9}{2}\frac{\mu_0\dot{\gamma}}{g\Delta\rho}\int_0^{\phi(0)}\frac{\hat{D}(\phi)}{f(\phi)}\,d\phi, \qquad [7]$$

where $z = h$ denotes the interface between the resuspended layer and the pure fluid, and $\phi(0)$ is the unknown particle concentration at $z = 0$, the bottom of the suspension. Equation [6] presupposes that the resuspension height never reaches the liquid–air interface. In addition, the requirement that the total particle volume be conserved gives

$$h_0\phi_0 = \int_0^h \phi\,dz, \qquad [8]$$

where h_0 denotes the initial height of the settled layer in the absence of shear.

By combining [7] and [8], we then find that

$$1 = \frac{A}{\phi_0}\int_0^{\phi(0)}\frac{\hat{D}(\phi)}{f(\phi)}\,d\phi, \qquad [9]$$

with

$$A = \frac{9}{2}\frac{\mu_0\dot{\gamma}}{g\Delta\rho h_0}, \qquad [10]$$

which shows that $\phi(0)$ is a monotonically decreasing function of the dimensionless parameter A, the latter representing the ratio between viscous and buoyancy forces. Finally, the ratio of the resuspension height h to the initial height of the settled layer h_0 can be obtained from [6] and [9]:

$$\frac{h - h_0}{h_0} = A\int_0^{\phi(0)}\frac{\hat{D}(\phi)}{f(\phi)}\left(\frac{1}{\phi} - \frac{1}{\phi_0}\right)d\phi, \qquad [11]$$

which clearly shows that, for a given choice of $f(\phi)$, $\hat{D}(\phi)$, $\mu_r(\phi)$ and ϕ_0, the relative change in the resuspension height is a function only of A. The dependence of h/h_0 on A can be easily approximated by studying its asymptotic behavior, noting that $h/h_0 \to 1$ as $A \to 0$. On the other hand, as $A \to \infty$, the suspension becomes infinitely dilute everywhere, provided that h remains below the liquid–air interface. Therefore, on letting $\phi \to 0$, which implies that $f(\phi) \to 1$, $\mu_r(\phi) \to 1$ and $\hat{D}(\phi) \to \phi^2/2$ on account of [2], [3] and [5], we obtain from [6] and [9], that $h/h_0 \to BA^{1/3}$ with $B = \frac{1}{4}(6\phi_0)^{2/3} \approx 0.574$. Finally, considering these asymptotic results, it is found that the simple interpolation formula

$$\frac{h}{h_0} = [1 + (BA^{1/3})^m]^{1/m} \quad \text{with} \quad m = 0.93 \qquad [12]$$

represents the values of h/h_0, as computed for the numerical solution of [9] and [11], to within a few percent over the whole range of A.

Since, according to our model, both the sedimentation velocity and the effective diffusivity are proportional to the square of the particle radius, it is not surprising that A, and therefore the relative resuspension height h/h_0, is independent of the particle dimension a; however, the time Δt needed to reach steady state,

$$\Delta t \propto \frac{(\Delta h)^2}{D} \propto \frac{A^{2/3} h_0^2}{\dot{\gamma} a^2} \qquad [13]$$

for $A \gg 1$, is strongly dependent on a.

3. EXPERIMENTAL TECHNIQUE AND RESULTS

3.1. Equipment and materials

The Couette device (see figure 1), made of transparent plexiglass, had inner and outer radii of 7.575 and 8.214 cm, respectively, giving a gap width of 0.639 cm, while a typical resuspension height was about 3–4 cm. A layer of mercury prevented the particles from migrating out of the gap as a result of shear-induced diffusion from regions of high shear to low (LA2). Since the gap-to-inner radius ratio was small, the shear was taken as being constant across the gap. The inner cylinder was driven by a variable speed motor and the shear rate across the gap could range from 4 to 143 s^{-1}, which was substantially below the critical shear rates required for the onset of Taylor instability (500–1000 s^{-1} in our case).

Two sizes of glass spheres and Diakon acrylic spheres were used in our experiments. The glass spheres were approx. 68 and 139 μm dia and had a density of 2.45 g/cm^3, while the Diakon spheres were approx. 387 μm dia and had a density of 1.19 g/cm^3. The Diakon spheres were density-segregated in a glycerin–water solution before they were used.

The two suspending liquids were mineral oil and Dow Corning 200 fluid, with densities 0.876 and 0.980 g/cm^3, and viscosities 1.43 and 4.71 P at 25°C, respectively. These fluids are Newtonian and have viscosities that are only mildly temperature dependent.

Four types of suspensions were studied: 139 μm glass spheres in mineral oil; 139 μm glass spheres in Dow Corning 200 fluid; 68 μm glass spheres in Dow Corning 200 fluid; and 387 μm Diakon particles in Dow Corning 200 fluid. The shear rate was varied from 4 to 143 s^{-1}, thereby creating a range of the parameter A from 0.01 to 10.

Finally, a thermoprobe in contact with the mercury was used to measure the instantaneous fluid temperature, and a Haake RV20/CV20 temperature-controlled viscometer was used to measure the fluid viscosity at each temperature.

3.2. Experimental procedure

Before starting the experiments, the suspension was sheared for approx. 2 days in order to mix the particles and drive out all the air bubbles.

In each subsequent experiment the suspension was sheared at a fixed rate. It was found that, upon shearing, an initially settled layer of particles expanded slowly with a sharp interface separating the suspension and the pure fluid. Steady state was achieved after a time interval Δt (cf. [13]) which varied from a few minutes to an hour.

After steady state was reached, the height of the resuspended layer was measured, to within 0.01 cm, with a cathetometer. The r.p.m. of the motor was measured with an optical digital tachometer to within $\pm 0.025\%$.

3.3. Results

Before making any measurements, it was necessary to ensure that a secondary flow did not exist within the test section of the Couette gap. To this end, the trajectory of a single neutrally buoyany sphere suspended in a pure fluid was monitored, but no measurable vertical displacement was found which could have been caused by a second flow. Also, as mentioned earlier, the calculated critical shear rates for the onset of a Taylor instability in a narrow gap Couette flow with the fluids used in this study were greater by at least a factor of 4 than the highest shear rate attainable in our equipment.

The experimental data for $\Delta h/h_0$ as a function of A are plotted in figure 2 and show excellent quantitative agreement with the theoretical predictions, even though there are no adjustable parameters in the model.

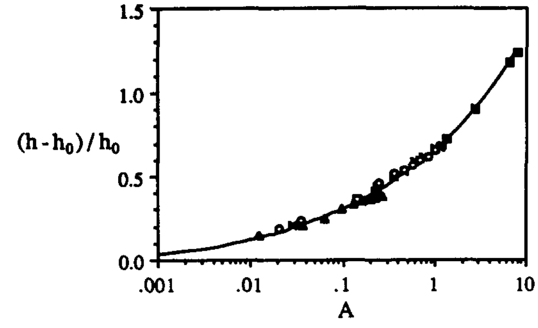

Figure 2. The height of the resuspended layer as a function of the parameter A: ——, theoretical prediction; ×, 68 μm glass beads in Dow Corning 200 fluid; ○, 139 μm glass beads in Dow Corning 200 fluid; △, 139 μm glass beads in mineral oil; ■, 387 μm Diakon beads in Dow Corning 200 fluid.

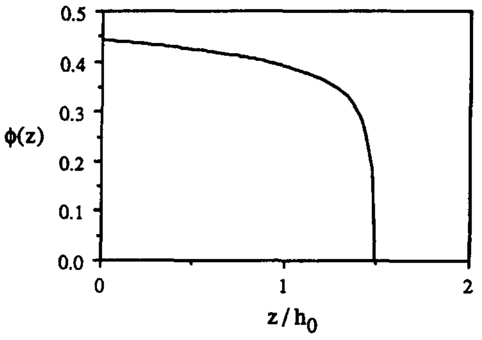

Figure 3. The theoretically determined concentration profile of 139 μm glass beads in Dow Corning 200 fluid at a shear rate of 52 s^{-1} ($A = 0.364$).

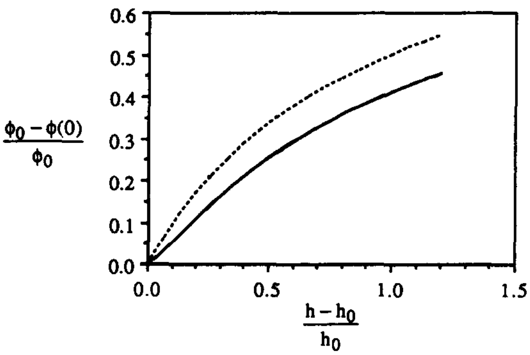

Figure 4. The particle concentration $\phi(0)$ at the bottom of the suspension as a function of the resuspension height h: ——, from the numerical evaluation of [9] and [11]; - - - -, assuming that the concentration profile is a step function.

It is worth noting that wall slip effects were undoubtedly negligibly small in these experiments since the particle-to-gap ratios, an important parameter for the existence of wall slip effects, were extremely small (<0.06) and the calculated particle concentrations were lower than 0.5 when $\Delta h/h_0$ exceeded 0.28. In addition, the measured resuspension heights for the 68 and 139 μm glass spheres were the same (as shown in figure 2), which also indicates that the effects of wall slip are negligible.

Finally, as was mentioned previously, the clear fluid and the resuspended layer were separated by a sharp interface. This is consistent with the model, in which the concentration $\phi = \phi(z)$ is predicted to be almost constant and equal to $\phi(0)$ throughout the suspension and to drop off sharply to zero very near the interface (see figure 3).

In fact, if the concentration profile were a step function with $\phi = \phi(0)$ for $0 < z < h$, then $\phi(0)$ would simply equal $h_0\phi_0/h$. As shown in figure 4, this simple expression provides a good approximation to the results of the theoretical model as obtained from the numerical evaluation of [9] and [11].

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